SEMESTRAL EXAMINATION
B. MATH III YEAR, I SEMESTER 2011-2012
DIFFERENTIAL EQUATIONS

Max. 100. Absolute time limit: 3hrs

The five questions carry a total of 110 marks. Answer as many as you can.

1. Show that the differential equation
$(1-x^2)y'' - xy' + 9y = 0$ on $(0, \infty)$ has two linearly independent power series solutions and that one of them is a polynomial. Evaluate the polynomial explicitly. [25]

2. Consider the Hermite polynomials $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}(e^{-x^2}), n = 0, 1, 2....$ Show that $e^{2xt-t^2} = \sum_{n=0}^{\infty} H_n(-x) \frac{(-1)^n}{n!} t^n$ for all real numbers x and t. [10]

3. Solve the initial value problem
$y'' + y' = 3x^2, y(0) = 0, y'(0) = 1, 0 \leq x < \infty$ explicitly using Laplace Transforms. [25]

4. Let $f_n(x) = \cos(n \cos^{-1}(x)), -1 < x < 1$. Show that f_n is a polynomial of degree n and that
$(1-x^2)f_n''(x) - xf_n'(x) + n^2 f_n(x) = 0.$ [25]

5. Find a set of solutions of the Heat Equation
$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial t}$ by separating the three variables. Also find a solution that satisfies the following properties:
a) $u(x, 0, t) = u(x, \pi, t) = 0$
b) $\lim_{t \to \infty} u(x, y, t) = \lim_{x \to -\infty} u(x, y, t) = 0.$
[There are many solutions. Do some guess work to write down one solution explicitly] [25]